A Note on the Seller’s Optimal Mechanism in Bilateral Trade with Two-Sided Incomplete Information*

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Abstract

It is shown with an example that, in bilateral trade problems with two-sided incomplete information, some seller types may obtain higher expected payoffs in mechanisms other than the one where they make a take-it-or-leave-it offer, contrary to popular belief. If one looks at the mechanism selection problem of the (informed) seller, then the optimality of a take-it-or-leave-it offer for the seller is restored. *Journal of Economic Literature* Classification Numbers: C72, C78, D82.

1 Introduction

Consider private-values bilateral trade problems with two-sided incomplete information. Some economists apparently believe that making a take-it-or-leave-it offer (hereafter this mechanism is denoted by $S$) is the “best” incentive compatible and individually rational mechanism for the seller.$^{1}$ This belief seems to be based on Riley and Zeckhauser [7], who showed that $S$

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$^{1}$See, e.g., Matthews and Postlewaite [4, p.243] and Palfrey and Srivastava [6, p.21].
is the seller’s optimal mechanism when there is no uncertainty about her valuation. I show with an example that, when the buyer does not know the seller’s valuation, some seller types may obtain higher expected payoffs in other mechanisms.\textsuperscript{2} The example may suggest that some seller types will select a mechanism other than $S$ when they can choose the mechanism. However, I show that this is not the case: $S$ is in fact the informed seller’s (i.e., the seller knows her type when she selects the mechanism) optimal mechanism. This result complements Riley and Zeckhauser [7], mentioned above, and Williams’ [9] result that $S$ is \textit{ex-ante} optimal when there is two-sided incomplete information.

The example compares the seller’s expected payoff in exogenously given mechanisms. The fact that $S$ is the informed seller’s optimal mechanism implies that seller types that obtain higher expected payoffs in mechanisms other than $S$ cannot use this to their advantage. The seller’s selection of one of these mechanisms will reveal some information about her type, which in turn, will alter the equilibria in that mechanism to her disadvantage. More specifically, in the example, low-valuation seller types do better when shifting from $S$ to a double auction. With incomplete information high-valuation buyers must bid sufficiently high to buy from high-valuation sellers, which raises the expected equilibrium price for low-valuation sellers. Thus low-valuation sellers prefer the double auction. However, when the seller chooses the mechanism, since high types prefer $S$ and separate themselves, the gain to lower types from the double auction is lost, and they will also use $S$.

The mechanism design problem of the informed principal was first studied by Myerson [5]. The setup in this note is similar to that of Maskin and Tirole [3], who analyze the principal-agent relationship when the principal has private information which does not directly affect the agent’s payoff. They show that the principal generically (in the space of payoff functions) does strictly better when she has private information. They also show that when players have quasi-linear utility functions (“the nongeneric case”) the principal’s utility does not depend on whether or not she has private information. The main result of this note, which also assumes quasi-linear utility, seems to be a special case of this last result: The informed seller’s optimal mechanism is $S$, which is, as Riley and Zeckhauser [7] showed, her optimal mechanism when her valuation is common knowledge. Actually, the analysis of Maskin

\textsuperscript{2}The results in this note are presented in terms of the seller, but since all of the arguments are symmetric, analogous results hold for the buyer.
and Tirole [3], as general as it is, does not apply to bilateral trade problems. They have many assumptions which are not satisfied in the standard bargaining environment studied here. For example, they assume that the type space is finite, the action is continuous, the utility functions of the players are strictly concave in the action, and there exists a feasible action and transfer that both parties prefer to the “null contract”. More importantly, their sorting assumption, which is central to their analysis, does not hold. This assumption is used for relating the solution of the principal’s mechanism selection problem to the Walrasian equilibria of the fictitious economy they construct and study. Therefore, their arguments cannot be extended to the setup of this note.

2 The Best Mechanism for the Seller

There is a seller \((i = s)\) who owns an indivisible good and a buyer \((i = b)\). Let \(V_i\) be \(i\)’s valuation (in dollars) of the good, which has a distribution \(F_i\) with the continuous density \(f_i\) that is positive over \([a_i, b_i]\). Both players are risk-neutral. Each player knows her valuation, but considers the other’s as a random variable. \(V_s\) and \(V_b\) are independent and \(v_i + \frac{F_i(v_i)}{f_i(v_i)}\) is increasing on \([a_i, b_i]\):

Let \(S\) denote the mechanism where the seller makes a take-it-or-leave-it offer, and \(D\) denote the \(\frac{1}{2}\)-double auction. Consider the example where both \(V_s\) and \(V_b\) are uniformly distributed on \([0, 1]\). The equilibria in \(S\) and \(D\) are:

\(S\): Type \(v_s\) seller offers \(\frac{1 + v_s}{2}\), type \(v_b\) buyer accepts the offer if \(v_b \geq \frac{1 + v_s}{2}\).

\(D\): Type \(v_s\) seller and type \(v_b\) buyer bid, respectively, \(\frac{2}{3} v_s + \frac{1}{4}\) and \(\frac{2}{3} v_b + \frac{1}{12}\).

Expected utilities of the seller and the buyer are:

\(S\): \(u_s^S(v_s) = \frac{1}{4}(1 - v_s)^2, u_b^S(v_b) = \begin{cases} 0 & v_b \leq \frac{3}{4} \\ \frac{1}{4}(2v_b - 1)^2 & v_b > \frac{3}{4} \end{cases}\)

\(D\): \(u_s^D(v_s) = \begin{cases} 0 & v_s \leq \frac{3}{4} \\ \frac{1}{4}(v_b - 1)^2 & v_b > \frac{3}{4} \end{cases}, u_b^D(v_b) = \begin{cases} 0 & v_b \leq \frac{1}{4} \\ \frac{1}{4}(v_b - 1)^2 & v_b > \frac{1}{4} \end{cases}\)

It is easy to see that,

\[u_s^S(v_s) \geq u_s^D(v_s) \text{ iff } v_s \geq c = \frac{1}{2} - \frac{1}{2 \sqrt{2}}.\]

\footnote{3Precisely, the last assumption is violated as long as the highest possible valuation of the seller is greater than the lowest possible valuation of the buyer.}

\footnote{4There is a continuum of equilibria in the double auction. See, e.g., Leininger, Linhard, and Radner [2] and Satterthwaite and Williams [8]. Here attention is restricted to the linear equilibrium which was first studied by Chatterjee and Samuelson [1].}
Hence, seller types with $v_s < c$ obtain higher expected payoffs in the $\frac{1}{2}$-double auction than what they would get by making a take-it-or-leave-it offer.

If the seller’s valuation for the good were common knowledge, then $S$ would maximize her payoff (Riley and Zeckhauser [7]). When there is uncertainty about the valuation of the seller, low-valuation seller types obtain higher expected payoffs in $D$. In $S$, the buyer’s strategy is independent of the uncertainty in the seller’s valuation: the buyer will accept the offer iff it is lower than her valuation. However, this is not the case in $D$. In particular, she bids more aggressively if it is more likely that the seller has a high valuation for the good. So, low-valuation sellers benefit from the positive externality arising from the existence of high-valuation types in $D$.

Now consider the informed seller’s mechanism selection problem. Suppose, only for now, that she has to choose $S$ or $D$. It is not true that seller types with $v_s > (\leq)c$ will choose $S$ ($D$). If this were the case, then the seller’s choice would reveal some information about her valuation, i.e., whether it is bigger or smaller than $c$. The buyer would shade her bid, in $D$, if she were to know that $v_s < c$, rendering the choice of $D$ suboptimal for some seller types with $v_s < c$. More generally, among all feasible mechanisms, all seller types will select a mechanism that is payoff equivalent to $S$:

**Proposition 1** Consider the case in which the informed seller selects the trading mechanism (which has to be incentive compatible and individually rational) in the bilateral-trade problem described above. Making a take-it-or-leave-it offer is optimal for all seller types.

**Proof.** Due to the principle of inscrutability (Myerson [5]) there is no loss of generality in assuming that all seller types will choose the same mechanism. Suppose that a seller type obtains an expected payoff in this mechanism which is less than what she would obtain from $S$; then she would select $S$ since the equilibrium in $S$ is insensitive to the belief of the buyer, a contradiction. Hence, all seller types will have payoffs at least as large as their payoffs from $S$. Williams [9] showed the ex-ante optimality of $S$ for the seller. One, then, needs only to show that a seller type getting a higher expected payoff than what she would get from $S$ contradicts Williams’ [9] result. So, suppose that a seller type ($v_s$) obtains a higher expected payoff than what

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5The principle of inscrutability essentially states that the principal (here the seller) does not need to reveal any information in her choice of a mechanism, she can always build it into the mechanism itself. See Myerson [5] for a complete presentation.
she would obtain from $S$. By a continuity argument, for sufficiently small $\epsilon > 0$, all seller types $v_s$ such that, $\max\{0, \pi_s - \epsilon\} < v_s < \min\{1, \pi_s + \epsilon\}$ will have higher expected payoffs than what they would obtain from $S$, as well. However, this contradicts the ex-ante optimality of $S$ for the seller as it was shown that all seller types would obtain payoffs at least as large as their payoffs from $S$. ■

References


