

TAX HARMONIZATION AND PARETO-EFFICIENCY

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Abstract

We consider the problem of tax harmonization among independent nations that have separate government budget constraints. The case for harmonization in this environment is shown to be weak, unless the nations do not trade with each other even when there is but one consumer in each country. We then contrast our findings to a related situation in which countries are not required to meet a balance of payments condition. Here, when each country has but a single consumer, a case for tax harmonization emerges.

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1. Introduction

Although tax-harmonization has been studied in the context of a fiscal federation¹—a central government sharing various tax and expenditure powers with state or provincial governments—it has received little formal analysis² in the context of the European Economic Union where there is no federal government with its own taxation powers. Any credible attempt at tax harmonization or coordination must be in the interests of each of the member states in order for such measures to be realised. Anecdotal evidence suggests that general opinion regards tax harmonization to be an improvement. Reasons range from the elimination of cross-border shopping (witness the recent abolition of duty-free shopping within the EEU) to the protection of member states from internal interest groups. In addition to these extremes, are arguments for tax harmonization on the grounds of efficiency³ and it is this question that we address in this paper.

The model we employ is too simple to capture the effects of cross-border shopping or those of powerful internal interest groups, but can establish whether or not tax harmonization entails efficiency gains. As for the model, there are two countries, each of which has a competitive production sector. There is free trade between them. The countries can, if they wish, implement different, destination based commodity taxes. There are two further elements of the model not always seen in the literature: there are several individuals in each country and each country has a real reason to implement commodity taxes, namely, to pay for a country specific public good. Thus, it is possible that not all Pareto optima are first best. Producers are exempt from taxation, and we focus primarily on the case in which countries impose no tariffs on imports or exports. Thus, there exists independent control over producer prices and consumer prices. These are precisely the circumstances under which world shadow prices are proportional to producer prices in the Diamond and Mirrlees (1971) framework. The fact that our

¹ See Blackorby and Brett (1999a) and the references there.

² An exception is Keen (1987, 1989) whose work is discussed in Section 5 below. We discuss a somewhat related strand of the literature contained in Turunen-Red and Woodland (1990, 1993, 1996) in Section 4.

³ See Keen (1987, 1989) and Turunen-Red and Woodland (1990, 1993, 1996).

results indicate a deviation from this principle is due to restrictions on international transfers, and not due to differences in the ways in which prices are modelled.

Using the tax-reform methodology of Guesnerie (1977, 1995), we argue that there does not appear to be an obvious claim for tax harmonization in the general model. Indeed, the shadow prices of commodities are shown to be different in the two countries. Then, to move closer to the existing literature, we drop the requirement of government budget balance in each country and repeat the above exercise. However, the case for tax harmonization remains elusive. Only when we specialize the model to the case of a single consumer in each country do we find a strong argument for tax harmonization, and then only when optimal lump sum transfers across nations are permitted. In such circumstances, international transfers are actually transfers among individuals. Thus, tax harmonization is optimal only when the second-best is first-best.

The remainder of the paper is organized as follows. Section 2 provides a description of the general model and derives some second best taxation rules. We introduce a notion of tax harmonization in Section 3. A discussion of Turunen-Red and Woodland (1990) is contained in Section 4. Single consumer results related to the work of Keen (1987, 1989) are presented in Section 5, and concluding remarks are found in Section 6. An appendix contains a discussion of price normalizations in the model.

2. The Two Country Model

The countries, α and β , provide a fixed amount of a public good for their respective citizens which is financed by country-specific, residence based commodity taxes, a one hundred percent profit tax and a demogrant (a poll tax or subsidy). We could avoid the issue of profit taxation by assuming constant returns to scale technologies. In general, therefore, the consumers in the two countries do not face the same prices even though there is free trade. We consider only the case of direct control of taxes as well as a country-specific demogrant. However, in passing, we also deal with optimal international transfers. Consumer prices are given by $q^\alpha = p + t^\alpha$ where t^α is the vector of country-specific taxes in country α ; for country β , the prices are $q^\beta = p + t^\beta$. The

governments may also provide their respective citizens with a poll subsidy or demogrant. We denote these quantities by m_α and m_β . The individual indirect utility functions⁴ are given by

$$u_h = V^h(q^\alpha, m_\alpha, g^\alpha) \quad \text{for all } h \in \mathcal{A} \quad (2.1)$$

and

$$u_h = V^h(q^\beta, m_\beta, g^\beta) \quad \text{for all } h \in \mathcal{B} \quad (2.2)$$

where \mathcal{A} (respectively \mathcal{B}) is the set of people living in country α (respectively β). Let H_α and H_β be the number people living in α and β respectively. The competitive production sectors are represented by profit functions,⁵ $\pi^\alpha(p)$ and $\pi^\beta(p)$, so that supply is given by

$$y^c = y^\alpha + y^\beta = \nabla_p \pi^\alpha(p) + \nabla_p \pi^\beta(p). \quad (2.3)$$

We assume that the technologies for producing the public goods are given by

$$g^\alpha \leq F^\alpha(y^{g^\alpha}) \quad \text{where } F^\alpha(0) = 0, \quad (2.4)$$

and

$$g^\beta \leq F^\beta(y^{g^\beta}) \quad \text{where } F^\beta(0) = 0. \quad (2.5)$$

F^α and F^β are assumed to be concave and differentiable.

We reserve the term shadow prices for a set of prices such that if the public firms were asked to minimize cost subject to these prices, they would choose the (second-best) optimal combination of inputs. Let η^α be the shadow prices in country α . Then, public enterprises in country α act as if they solve the program

$$\min_{y^{g^\alpha}} \{ \eta^{\alpha T} y^{g^\alpha} \mid F^\alpha(y^{g^\alpha}) \geq g^\alpha \}. \quad (2.6)$$

The first order conditions associated with this problem are given by

$$\eta^{\alpha T} = \lambda^\alpha \nabla_{y^\alpha}^T F^\alpha \quad (2.7)$$

⁴ We assume that these are differentially strongly quasi-convex; see Blackorby and Diewert (1979) for details.

⁵ These are assumed to be differentially strongly convex; see Diewert, Avriel, and Zang (1981) for details. Given that the production sector is competitive, there is no loss of generality in assuming there to be but one firm in each country; see page 68 in Bliss (1975).

for some positive scalar λ^α . A similar argument can be made for country β . We do not assume in advance that there is production efficiency, that is, we do not assume that producer prices are the social shadow prices to be given to the managers of the public firms.

Net supply is given by

$$\tilde{y}^* = \tilde{y}^{*c} - y^{g\alpha} - y^{g\beta}. \quad (2.8)$$

Equilibrium is said to exist when

$$-\tilde{x}^* + \tilde{y}^* \geq 0 \quad (2.9)$$

where

$$\tilde{x}^* = \tilde{x}^{*\alpha} + \tilde{x}^{*\beta} = \sum_{h \in \mathcal{A}} \tilde{x}^{*h} + \sum_{h \in \mathcal{B}} \tilde{x}^{*h}. \quad (2.10)$$

In order to close the model, each country has a hundred percent profit taxation and must balance their respective budgets so that

$$\pi^\alpha(p) + t^{\alpha T} \tilde{x}^{*\alpha} = p^T y^{g\alpha} + H_\alpha m_\alpha \quad (2.11)$$

and

$$\pi^\beta(p) + t^{\beta T} \tilde{x}^{*\beta} = p^T y^{g\beta} + H_\beta m_\beta. \quad (2.12)$$

In addition, the countries must maintain balance of payments equilibrium,

$$(y^{\alpha T} - y^{g\alpha T} - \tilde{x}^{*\alpha T})p = 0 \quad (2.13)$$

and

$$(y^{\beta T} - y^{g\beta T} - \tilde{x}^{*\beta T})p = 0. \quad (2.14)$$

By Walras' Law, the budgets of the two countries taken together must be balanced. It also the case that in a tight equilibrium—when (2.9) holds with equality—any one of (2.11)—(2.14) implies the other three.

Proposition 2.1: *Given a tight equilibrium, (2.9) holds with equality, any one of (2.11)—(2.14) implies the other three.*

Proof: We show that (2.11) implies (2.12)—(2.14); the others claims are similar. To verify this assertion, substitute (2.8) and (2.9) into (2.11), yielding

$$\pi^\alpha(p) + t^{\alpha T} \ddot{x}^\alpha = p^T (\dot{y}^c - \ddot{x} - y^{g\beta}) + H_\alpha m_\alpha. \quad (2.15)$$

Rearranging (2.15), using the definition of the tax rates, implies

$$t^{\alpha T} \ddot{x}^\alpha = \pi^\beta(p) - (q^\alpha - t^\alpha)^T \ddot{x}^\alpha - (q^\beta - t^\beta)^T \ddot{x}^\beta - p^T y^{g\beta} + H_\alpha m_\alpha. \quad (2.16)$$

Equivalently,

$$t^{\alpha T} \ddot{x}^\alpha = \pi^\beta(p) + (H_\alpha m_\alpha - q^{\alpha T} \ddot{x}^\alpha) + t^{\alpha T} \ddot{x}^\alpha + t^{\beta T} \ddot{x}^\beta - p^T y^{g\beta} - q^{\beta T} \ddot{x}^\beta. \quad (2.17)$$

The term in parentheses on the right hand side of (2.17) is zero, because consumers in country α exhaust their budgets. Thus,

$$\pi^\beta(p) + t^{\beta T} \ddot{x}^\beta - (p^T y^{g\beta} + H_\beta m_\beta) = 0, \quad (2.18)$$

which is equivalent to (2.12). Now, substituting the budget constraints for consumers in country β into (2.18) implies

$$\pi^\beta(p) + t^{\beta T} \ddot{x}^\beta - p^T y^{g\beta} + (p^T + t^{\beta T}) \ddot{x}^\beta = 0, \quad (2.19)$$

and, by the definition of $\pi^\beta(p)$,

$$p^T (\dot{y}^\beta - y^{g\beta} - \ddot{x}^\beta) = 0, \quad (2.20)$$

which is (2.14). Repeating this argument for country α shows that (2.11) implies (2.13). ■

This means that there are several different but equivalent ways to proceed. One could eliminate one of the equilibrium conditions and use any two of (2.11)—(2.14) or all of the equilibrium conditions and any one of (2.11)—(2.14). In any case a certain amount of asymmetry is present. We use all of the equilibrium conditions, (2.9), and the government budget constraint of country α , (2.11), to analyse the model.

The above model is written so that there are no international transfer payments. Much of the literature depends upon the claim that international transfers are available

as tax instruments and in addition may be chosen optimally. To address this extension of our model one only need change the constraints, (2.11)—(2.14) as follows.

$$\pi^\alpha(p) + t^{\alpha T} \ddot{x}^\alpha = p^T y^{g^\alpha} + H_\alpha m_\alpha + T \quad (2.21)$$

and

$$\pi^\beta(p) + t^{\beta T} \ddot{x}^\beta = p^T y^{g^\beta} + H_\beta m_\beta - T. \quad (2.22)$$

Equivalently,

$$(y^{\alpha T} - y^{g^{\alpha T}} - \ddot{x}^{\alpha T})p = T, \quad (2.23)$$

and

$$(y^{\beta T} - y^{g^{\beta T}} - \ddot{x}^{\beta T})p = -T. \quad (2.24)$$

It is immediate that the above proposition holds with this change in the model.

Utility increases are given by

$$\begin{aligned} du_h > 0 &\iff \nabla_{q^\alpha}^T V^h(q^\alpha, m_\alpha, g^\alpha) dp + \nabla_{q^\alpha}^T V^h(q^\alpha, m_\alpha, g^\alpha) d\tau^\alpha + \nabla_{m_\alpha} V^h(q^\alpha, m_\alpha, g^\alpha) dm_\alpha > 0 \\ &\iff -\ddot{x}^{hT} dp - \ddot{x}^{hT} d\tau^\alpha + dm_\alpha > 0 \quad \text{for } h \in \mathcal{A} \end{aligned} \quad (2.25)$$

and

$$\begin{aligned} du_h > 0 &\iff \nabla_{q^\beta}^T V^h(q^\beta, m_\beta, g^\beta) dp + \nabla_{q^\beta}^T V^h(q^\beta, m_\beta, g^\beta) d\tau^\beta + \nabla_{m_\beta} V^h(q^\beta, m_\beta, g^\beta) dm_\beta > 0 \\ &\iff -\ddot{x}^{hT} dp - \ddot{x}^{hT} d\tau^\beta + dm_\beta > 0 \quad \text{for } h \in \mathcal{B}. \end{aligned} \quad (2.26)$$

Let the matrices of demands be given by

$$\ddot{X}^{\alpha T} = \begin{bmatrix} \ddot{x}^{1\alpha T} \\ \ddot{x}^{2\alpha T} \\ \vdots \\ \ddot{x}^{H_\alpha T} \end{bmatrix} \quad \text{and} \quad \ddot{X}^{\beta T} = \begin{bmatrix} \ddot{x}^{1\beta T} \\ \ddot{x}^{2\beta T} \\ \vdots \\ \ddot{x}^{H_\beta T} \end{bmatrix}. \quad (2.27)$$

Starting from a tight equilibrium, and assuming that each government has as instruments its commodity tax vector and its demogrant, there exist strict Pareto-improving directions of change if and only if A has a solution where A is given by

$$\begin{aligned}
& \begin{bmatrix} -\dot{X}^{\alpha T} & -\dot{X}^{\alpha T} & 0_{H_\alpha \times N} & 1_{H_\alpha} & 0_{H_\alpha} & 0_{H_\alpha \times N} & 0_{H_\alpha \times N} \\ -\dot{X}^{\beta T} & 0_{H_\beta \times N} & -\dot{X}^{\beta T} & 0_{H_\beta} & 1_{H_\beta} & 0_{H_\beta \times N} & 0_{H_\beta \times N} \end{bmatrix} \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm_\alpha \\ dm_\beta \\ dy^{g\alpha} \\ dy^{g\beta} \end{bmatrix} \gg 0 \\
& \begin{bmatrix} -\nabla_q \dot{x}^* + \nabla_p \dot{y}^c & -\nabla_q \dot{x}^{\alpha} & -\nabla_q \dot{x}^{\beta} & -\nabla_{m_\alpha} \dot{x}^{\alpha} & -\nabla_{m_\beta} \dot{x}^{\beta} & -I_{N \times N} & -I_{N \times N} \\ 0_N^T & 0_N^T & 0_N^T & 0 & 0 & \nabla_{y^g}^T F^\alpha & 0_N^T \\ 0_N^T & 0_N^T & 0_N^T & 0 & 0 & 0_N^T & \nabla_{y^g}^T F^\beta \end{bmatrix} \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm_\alpha \\ dm_\beta \\ dy^{g\alpha} \\ dy^{g\beta} \end{bmatrix} \geq 0 \\
& \begin{bmatrix} \dot{y}^{\alpha T} + t^{\alpha T} \nabla_q \dot{x}^{\alpha} - y^{g\alpha T} & t^{\alpha T} \nabla_q \dot{x}^{\alpha} + \dot{x}^{\alpha T} & 0_N^T & t^{\alpha T} \nabla_{m_\alpha} \dot{x}^{\alpha} - H_\alpha & 0 & -p^T & 0_N^T \end{bmatrix} \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm_\alpha \\ dm_\beta \\ dy^{g\alpha} \\ dy^{g\beta} \end{bmatrix} = 0
\end{aligned} \tag{2.28}$$

The top line of (2.28) describes the strictly Pareto-improving directions of policy reform; the second line, the equilibrium-preserving directions. Directions satisfying the final equality in (2.28) preserve government budget balance, and hence, balance of payments equilibrium.

If there is no solution to (2.28), then the economy must be at a Pareto-optimum. By Motzkin's theorem,⁶ A has a solution if and only if B does not have a solution; B is given by

$$-v^T + r_\beta \nabla_{y^g}^T F^\beta = 0, \tag{2.29}$$

$$-v^T - z_\alpha p^T + r_\alpha \nabla_{y^g}^T F^\alpha = 0, \tag{2.30}$$

$$\sum_{h \in \mathcal{B}} s_h = v^T \nabla_m \dot{x}^{\beta}, \tag{2.31}$$

⁶ For a statement of Motzkin's theorem, see Mangasarian (1969, pp. 28–29).

$$\sum_{h \in \mathcal{A}} s_h = v^T \nabla_m \bar{x}^{\alpha} - z_{\alpha} (t^{\alpha T} \nabla_m \bar{x}^{\alpha} - H_{\alpha}), \quad (2.32)$$

and

$$\sum_{h \in \mathcal{B}} s_h \bar{x}^{hT} = -v^T \nabla_q \bar{x}^{\beta}, \quad (2.33)$$

$$\sum_{h \in \mathcal{A}} s_h \bar{x}^{hT} = -v^T \nabla_q \bar{x}^{\alpha} + z_{\alpha} (t^{\alpha T} \nabla_q \bar{x}^{\alpha} + \bar{x}^{\alpha T}), \quad (2.34)$$

$$\sum_{h=1}^H s_h \bar{x}^{hT} = -v^T \nabla_q \bar{x} + v^T \nabla_p \bar{y} + z_{\alpha} (\bar{y}^{\alpha T} - y^{g_{\alpha T}} + t^{\alpha T} \nabla_q \bar{x}^{\alpha}), \quad (2.35)$$

where

$$0 \neq s \geq 0, v \geq 0, \quad \text{and} \quad r \geq 0. \quad (2.36)$$

The vector s may be interpreted as the social marginal utilities of the various individuals' incomes. The components of v are the marginal social valuations of commodities to the economic community as a whole.⁷ We now set out in detail the qualitative features of the second-best optima.

Proposition 2.2: *At almost all Pareto-optima, if net trade in some commodity is non-zero, the social value of commodities is not equal to the producer prices, that is, if net trade is non-zero, there is production inefficiency in the world economy.*

Proof: First, subtract (2.33) and (2.34) from (2.35) to obtain

$$0 = v^T \nabla_p \bar{y} + z_{\alpha} (\bar{y}^{\alpha T} - y^{g_{\alpha T}} - \bar{x}^{\alpha T}). \quad (2.37)$$

Post-multiply (2.37) by p and use (2.13) to obtain

$$0 = z_{\alpha} \times [0]. \quad (2.38)$$

This implies that there are, in general, no restrictions on z_{α} , the multiplier on the country-specific budget constraint. In particular, it is usually non-zero. Thus, the constraint is almost always strictly binding.

Now suppose that there is net trade in some commodities. This implies that the vector contained in parentheses on the right hand side of (2.37) is non-zero. Combined

⁷ This is a standard argument; see Guesnerie (1995) or Myles (1995).

with a non-zero value of z_α , means that in general the social values of commodities are not equal to producer prices, i.e.,

$$v \neq \mu p. \quad (2.39)$$

Otherwise, homogeneity of supply requires the first term on the right hand side of (2.37) to vanish. This yields a contradiction. ■

Not only are the producer prices not the world shadow prices, but the shadow prices are not the same within the countries.

Proposition 2.3: *If one of the government budget constraints or balance of payments constraints is strictly binding, then the shadow prices in the two countries are not the same.*

Proof: By (2.29) and (2.30)

$$\nabla_{y^\alpha}^T F^\alpha(y^{g_\alpha}) = \frac{v}{r_\beta}; \quad \nabla_{y^\beta}^T F^\beta(y^{g_\beta}) = \frac{v + z_\alpha p}{r_a}. \quad (2.40)$$

Thus, shadow prices are the same (up to scalar multiplication) if and only if $z_\alpha = 0$. The existence of the separate government budget constraints leads to a divergence of shadow prices in the two countries. ■

Because there are separate budget constraints for the two countries, the second-best Pareto-optima are not production efficient from the point of view of the world economy. It is a simple corollary of the above results that if lump-sum transfer payments were permitted between countries and if they were chosen efficiently, then the shadow prices in the two countries would be proportional to each other and proportional to producer prices, that is, there would be production efficiency.

Proposition 2.4: *If international-transfers are permitted, then, at all second-best Pareto-optima, shadow prices are proportional to producer prices, that is, there is production efficiency in the world economy.*

Proof: If T is chosen optimally, then z_α is identically zero. From (2.29), (2.30), and (2.37) it follows that the social shadow prices are proportional to producer prices. ■

The above analysis of the set of second-best Pareto optima also provides immediate insight into the nature of strict Pareto-improving tax reforms for those equilibria that are not Pareto-optimal. More specifically, there are strict Pareto-improving tax reforms if and only if the system A above has a solution. By Motzkin's Theorem, if A has a solution then there is no solution to B satisfying the stated inequality conditions on the multipliers. Note also that among these strict Pareto-improving reforms that a non negligible subset of them involve temporary inefficiencies in production.⁸

3. Tax Harmonization

The general model outlined above can be easily modified to allow for restrictions on the tax powers of the two governments. Harmonization is one such restriction. In its extreme form, harmonization entails equality of tax rates in the two countries.⁹ In this section, we explore the consequences of such restrictions on second-best optima. Suppose, that there exists some target tax rate, \bar{t}^* , that would be optimal for both countries. If such a target tax vector exists, then, at the second-best Pareto optima, constraining the country specific taxes to be equal to this target should not be binding. We write these constraints as

$$dt^\alpha = \bar{t}^* - t^\alpha \tag{3.1}$$

and

$$dt^\beta = \bar{t}^* - t^\beta. \tag{3.2}$$

If such a target exists, then it must be the case that

$$dt^\alpha - dt^\beta = t^\alpha - t^\beta. \tag{3.3}$$

⁸ See Guesnerie (1977,1995) for a discussion.

⁹ Harmonization on a subset of taxes could be modeled by restricting changes in some components of the tax vector to be identical.

If this is not satisfied, then it must be the case that no such \tilde{t}^* exists. We note that the directions of tax reform studied here belong to the category of shifts toward a common target, as analyzed by Keen (1987) and Turunen–Red and Woodland (1990).

Adding these constraints to the previous system, A , yields a nonhomogeneous system of equations that can be converted into the homogeneous system, called \bar{A} , below. There exist strict Pareto-improving directions of change if and only if \bar{A} has a solution where \bar{A} is given by

$$\begin{bmatrix} -\tilde{X}^{\alpha T} & -\tilde{X}^{\alpha T} & 0_{H_\alpha \times N} & 1_{H_\alpha} & 0_{H_\alpha} & 0_{H_\alpha \times N} & 0_{H_\alpha \times N} & 0_{H_\alpha} \\ -\tilde{X}^{\beta T} & 0_{H_\beta \times N} & -\tilde{X}^{\beta T} & 0_{H_\beta} & 1_{H_\beta} & 0_{H_\beta \times N} & 0_{H_\beta \times N} & 0_{H_\beta} \\ 0_N^T & 0_N^T & 0_N^T & 0 & 0 & 0_N^T & 0_N^T & 1 \end{bmatrix} \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm_\alpha \\ dm_\beta \\ dy^{g\alpha} \\ dy^{g\beta} \\ d\eta \end{bmatrix} \gg 0, \quad (3.4)$$

$$\begin{bmatrix} -\nabla_q \tilde{x} + \nabla_p \tilde{y}^c & -\nabla_q \tilde{x}^\alpha & -\nabla_q \tilde{x}^\beta & -\nabla_{m_\alpha} \tilde{x}^\alpha & -\nabla_{m_\beta} \tilde{x}^\beta & -I_{N \times N} & -I_{N \times N} & 0_N \\ 0_N^T & 0_N^T & 0_N^T & 0 & 0 & \nabla_{y^g}^T F^\alpha & 0_N^T & 0 \\ 0_N^T & 0_N^T & 0_N^T & 0 & 0 & 0_N^T & \nabla_{y^g}^T F^\beta & 0 \\ 0_N^T & 0_N^T & 0_N^T & 0 & 0 & 0_N^T & 0_N^T & 0 \end{bmatrix} \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm_\alpha \\ dm_\beta \\ dy^{g\alpha} \\ dy^{g\beta} \\ d\eta \end{bmatrix} \geq 0, \quad (3.5)$$

and

$$[C_1 \quad C_2] \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm_\alpha \\ dm_\beta \\ dy^{g\alpha} \\ dy^{g\beta} \\ d\eta \end{bmatrix} = 0 \quad (3.6)$$

where

$$C_1 = \begin{bmatrix} \tilde{y}^{\alpha T} + t^{\alpha T} \nabla_q \tilde{x}^\alpha - y^{g\alpha T} & t^{\alpha T} \nabla_q \tilde{x}^\alpha + \tilde{x}^{\alpha T} & 0_N^T \\ 0_{N \times N} & I_N & -I_N \end{bmatrix} \quad (3.7)$$

and

$$C_2 = \begin{bmatrix} t^{\alpha T} \nabla_m \tilde{x}^{\alpha} - H_{\alpha} & 0 & -p^T & 0_N^T & 0 \\ 0_N & 0_N & 0_{N \times N} & 0_{N \times N} & t^{\alpha} - t^{\beta} \end{bmatrix}. \quad (3.8)$$

A feasible strictly Pareto-improving direction of harmonizing tax reforms exists if and only if there exists a solution to \bar{A} . Applying Motzkin's theorem, there are no feasible strictly Pareto-improving directions of harmonizing tax reform when the following set of equations, called \bar{B} , has a solution.

$$s_{\eta} + w^T (t^{\alpha} - t^{\beta}) = 0, \quad (3.9)$$

$$-v^T + r_{\beta} \nabla_{y^g} F^{\beta} = 0, \quad (3.10)$$

$$-v^T - z_{\alpha} p^T + r_{\alpha} \nabla_{y^g} F^{\alpha} = 0, \quad (3.11)$$

$$\sum_{h \in \mathcal{B}} s_h = v^T \nabla_m \tilde{x}^{\beta}, \quad (3.12)$$

$$\sum_{h \in \mathcal{A}} s_h = v^T \nabla_m \tilde{x}^{\alpha} - z_{\alpha} (t^{\alpha T} \nabla_m \tilde{x}^{\alpha} - H_{\alpha}), \quad (3.13)$$

and

$$\sum_{h \in \mathcal{B}} s_h \tilde{x}^{*hT} = -v^T \nabla_q \tilde{x}^{\beta} - w^T, \quad (3.14)$$

$$\sum_{h \in \mathcal{A}} s_h \tilde{x}^{*hT} = -v^T \nabla_q \tilde{x}^{\alpha} + z_{\alpha} (t^{\alpha T} \nabla_q \tilde{x}^{\alpha} + \tilde{x}^{*\alpha T}) + w^T, \quad (3.15)$$

$$\sum_{h=1}^H s_h \tilde{x}^{*hT} = -v^T \nabla_q \tilde{x}^* + v^T \nabla_p \tilde{y}^* + z_{\alpha} (\tilde{y}^{*\alpha} - y^{g\alpha T} + t^{\alpha T} \nabla_q \tilde{x}^{\alpha}), \quad (3.16)$$

where

$$0 \neq (s_1, \dots, s_H, s_{\eta}) \geq 0, v \geq 0, \quad \text{and} \quad r \geq 0. \quad (3.17)$$

The only differences between this system and the characterization presented in equations (2.29)–(2.36) of the last section is the introduction of w , an N -dimensional vector of multipliers, in equations (3.14) and (3.15), and the addition of (3.9). The following proposition shows that the harmonization constraint is almost always binding and hence that there does not exist a target tax rate, \tilde{t}^* , which is Pareto optimal.

Proposition 3.1: *With or without optimal international transfers, Pareto optimality implies that $t^{\alpha} \neq t^{\beta}$ almost everywhere.*

Proof: We need only to show that $w \neq 0$. First post-multiply (3.14) by q^β , multiply (3.12) by $-m_\beta$, and add the resulting expressions to yield

$$w^T q^\beta = 0. \quad (3.18)$$

Thus, the vector of multipliers is orthogonal to original consumer prices in country β . Performing analogous calculations on (3.15) and (3.13) gives

$$z_\alpha (\bar{x}^{\alpha T} q^\alpha - H_\alpha m_\alpha) + w^T q^\alpha = 0. \quad (3.19)$$

But the term in parentheses in (3.19) is zero, because all consumers in country α exhaust their budgets. Thus,

$$w^T q^\alpha = 0. \quad (3.20)$$

The vector of multipliers on the harmonization constraints is also orthogonal to consumer prices in country α . Moreover,

$$w^T (q^\alpha - q^\beta) = w^T (t^\alpha - t^\beta) = 0 \quad (3.21)$$

which in turn implies that $s_\eta = 0$. Thus, if taxes differ between the two countries, some components of w must be non-zero, so that harmonized movements in taxation impose a real restriction on tax power. If taxes are already identical, equation (3.21) imposes no restrictions on w . However, (3.18), which is now identical to (3.20), continues to impose restrictions. In general, therefore, w has non-zero components, so that the harmonization constraints are almost always strictly binding. ■

This is in stark contrast to the existing literature in which tax harmonizations are strict Pareto improvements. In order to explore this difference we summarize the results of the two previous sections. The solutions to the system of equations B describes all Pareto-optima in the model. If B has no solution then there exists a solution to the system A , that is, there are feasible strictly Pareto-improving directions of tax reform. If the system \bar{A} has no solution then there are no feasible strictly Pareto-improving direction of tax harmonization. In this case the system \bar{B} must have a solution and we have shown that this solution entails non zero multipliers on the tax harmonization constraints, i.e., $w \neq 0$. However, if \bar{B} has a solution with a non zero w , then the system B does not have a solution and hence system A does have a solution. That is, there

exist feasible strictly Pareto-improving directions of tax reform but none of these entail tax harmonization of the form we study. The next two sections explore the reasons why our results are at odds with the existent literature.

4. Related Literature

Turunen-Red and Woodland (1990) investigate similar issues in a somewhat different model; making the necessary changes in notation, we sketch their results in our context. We make some modifications to their framework: there are several consumers in each country, there is only a demogrant in each country, and governments have something upon which to spend their money—a public good. In addition to the commodity taxes, both firms and consumers must pay tariffs, τ^α and τ^β , which are, however, fixed.¹⁰ Thus producer prices are $p^\alpha = p + \tau^\alpha$ and $p^\beta = p + \tau^\beta$ and consumer prices are $q^\alpha = p + \tau^\alpha + t^\alpha$ and $q^\beta = p + \tau^\beta + t^\beta$ respectively. Because governments collect tariff revenue on the difference between domestic consumption and domestic production, the budget constraint for country α becomes

$$\pi^\alpha(p^\alpha) + t^{\alpha T} \dot{x}^\alpha + \tau^{\alpha T} (\dot{x}^\alpha - \dot{y}^\alpha) = p^T y^{g\alpha} + H_\alpha m_\alpha. \quad (4.1)$$

The second-best in this model is characterized by the existence of a solution to the following equations.

$$-v^T + r_\beta \nabla_{y^g} F^\beta = 0, \quad (4.2)$$

$$-v^T - z_\alpha p^T + r_\alpha \nabla_{y^g} F^\alpha = 0, \quad (4.3)$$

$$\sum_{h \in \mathcal{B}} s_h = v^T \nabla_m \dot{x}^\beta, \quad (4.4)$$

$$\sum_{h \in \mathcal{A}} s_h = v^T \nabla_m \dot{x}^\alpha - z_\alpha \left((\tau^{\alpha T} + t^{\alpha T}) \nabla_m \dot{x}^\alpha - H_\alpha \right), \quad (4.5)$$

and

$$\sum_{h \in \mathcal{B}} s_h \dot{x}^{hT} = -v^T \nabla_q \dot{x}^\beta, \quad (4.6)$$

$$\sum_{h \in \mathcal{A}} s_h \dot{x}^{hT} = -v^T \nabla_q \dot{x}^\alpha + z_\alpha \left((\tau^{\alpha T} + t^{\alpha T}) \nabla_q \dot{x}^\alpha + \dot{x}^{\alpha T} \right), \quad (4.7)$$

¹⁰ If the tariffs are being chosen optimally, then some additional difficulties arise; see Blackorby and Brett (1999b).

$$\sum_{h=1}^H s_h \bar{x}^{hT} = -v^T \nabla_q \bar{x}^* + v^T \nabla_p \bar{y}^* + z_\alpha (\bar{y}^{*\alpha} - y^{g\alpha T} + (\tau^{\alpha T} + t^{\alpha T}) \nabla_q \bar{x}^{*\alpha} - \tau^{\alpha T} \nabla_p \bar{y}^{*\alpha}), \quad (4.8)$$

where

$$0 \neq s \geq 0, v \geq 0, \quad \text{and} \quad r \geq 0. \quad (4.9)$$

The differences between these equations and those presented in Section 2 of this paper reflect the effects of changes in taxes and income on the value of tariff collections.

Subtracting (4.6) and (4.7) from (4.8), yields

$$0 = v^T \nabla_p \bar{y}^{*\alpha} + v^T \nabla_p \bar{y}^{*\beta} + z_\alpha [\bar{y}^{*\alpha} - y^{g\alpha T} - \bar{x}^{*\alpha} - \tau^{\alpha T} \nabla_p \bar{y}^{*\alpha}]. \quad (4.10)$$

Note that without optimal international transfers, the shadow prices are in general different in the two countries, and that, even if there are optimal international transfers, $z_\alpha = 0$, the shadow prices are not proportional to producer prices. In fact, even if the tariffs in the two countries were the same, as might be the case if the countries were inside a customs union, and $z_\alpha = 0$, the shadow prices are proportional to producer prices plus the common tariff barrier. Next, repeating the argument of the previous section, it is immediately apparent that the argument for tax harmonization is if anything even weaker in this context.

Turunen–Red and Woodland (1990) confine their attention to deriving a set of conditions under which $z_\alpha = 0$. These are the many-country analogue of our net trade condition, and a condition ruling out price responses in consumption. We note, using the argument set out in the appendix, that one cannot freely normalize one producer to be equal to one as done by Turunen-Red and Woodland (1990). This makes their condition, which is stated in terms of normalized prices, somewhat difficult to interpret.

5. Some Single Consumer Results

The existing literature on tax harmonization, for example, the work of Keen (1987, 1989),¹¹ differs from the above in two ways. First of all, there is but one consumer in each country. Second, the international transfers are chosen optimally. Under these

¹¹ The work of and Turunen-Red and Woodland (1990) was discussed in the previous section. Their related papers, (1993, 1996), are based on the same single-consumer model.

assumptions, there are no commodity taxes at the second-best optima. That is, the second-best is first-best. If either of these two assumptions fails, then commodity taxes are required at all Pareto-optima and tax harmonization is not optimal. We illustrate the first best nature of much of the existing literature by considering some special cases of our results.

First assume that there is but one consumer in each country.¹² Rewriting \bar{B} in this case yields

$$s_\eta + w^T (t^\alpha - t^\beta) = 0, \quad (5.1)$$

$$-v^T + r_\beta \nabla_{y^g} F^\beta = 0, \quad (5.2)$$

$$-v^T - z_\alpha p^T + r_\alpha \nabla_{y^g} F^\alpha = 0, \quad (5.3)$$

$$s_\beta = v^T \nabla_m \bar{x}^{\beta}, \quad (5.4)$$

$$s_\alpha = v^T \nabla_m \bar{x}^{\alpha} - z_\alpha (t^{\alpha T} \nabla_m \bar{x}^{\alpha} - 1), \quad (5.5)$$

and

$$s_\beta \bar{x}^{\beta T} = -v^T \nabla_q \bar{x}^{\beta} - w^T, \quad (5.6)$$

$$s_\alpha \bar{x}^{\alpha T} = -v^T \nabla_q \bar{x}^{\alpha} + z_\alpha (t^{\alpha T} \nabla_q \bar{x}^{\alpha} + \bar{x}^{\alpha}) + w^T, \quad (5.7)$$

$$s_\alpha \bar{x}^{\alpha T} + s_\beta \bar{x}^{\beta T} = -v^T \nabla_q \bar{x} + v^T \nabla_p \bar{y} + z_\alpha (\bar{y}^{\alpha} - y^{g\alpha T} + t^{\alpha T} \nabla_q \bar{x}^{\alpha}), \quad (5.8)$$

where

$$0 \neq s \geq 0, v \geq 0, \quad \text{and} \quad r \geq 0. \quad (5.9)$$

Even in the case of one consumer per country there is no change in the results if there are no international transfers.

Using the Slutsky equation and (5.4) and (5.6) yields

$$-v^T \nabla_{qq} E^\beta(u_\beta, q^\beta) = w^T. \quad (5.10)$$

Using (5.5) and (5.7) along with the Slutsky equation yields

$$(v^T - z_\alpha t^{\alpha T}) \nabla_{qq} E^\alpha(u_\alpha, q^\alpha) = w^T. \quad (5.11)$$

¹² This is the same as assuming that there are many consumers but that each government has the power and information to engage in individual lump-sum transfers.

In general, the multipliers on the tax harmonization constraints, w , are nonzero. That is, even with only one consumer in each economy, tax harmonization is not a characteristic of the Pareto-optima. Now suppose that the international transfer are chosen optimally. This implies that $z_\alpha = 0$. Adding (5.10) and (5.11) yields

$$v^T \left(\nabla_{qq} E^\alpha(u_\alpha, q^\alpha) + \nabla_{qq} E^\beta(u_\beta, q^\beta) \right) = 0. \quad (5.12)$$

Thus v is the zero eigen-vector of the Hessian of $E^\alpha + E^\beta$. Now suppose that $t^\alpha = t^\beta = \bar{t}^*$. Then (5.12) implies that

$$v = \mu(p + \bar{t}^*), \quad \mu > 0. \quad (5.13)$$

Next, subtracting (5.6) and (5.7) from (5.8), and using the fact that $z_\alpha = 0$ yields

$$v^T \nabla_p \bar{y}^* = 0. \quad (5.14)$$

As the profit function is strongly convex, this implies that

$$v = \theta p, \quad \theta > 0. \quad (5.15)$$

In the appendix it is shown that it is possible to normalize $t_1^\alpha = t_1^\beta = 0$ and $p_1 = 1$ without loss of generality. The top line of (5.13) then implies $\mu = 1$ and the top line of (5.15) shows that $\theta = 1$. Taken together, these imply that $\bar{t}^* = 0$. That is, tax harmonization is optimal only if the optimal commodity tax vector is zero. Thus producer prices are equal to consumer prices and the only Pareto-optima are first-best.

The converse is also true. Suppose that $w = 0$. Then, it follows that

$$v = \mu_\beta(p + t^\beta) \quad (5.16)$$

and

$$v - z_\alpha t^\alpha = \mu_\alpha(p + t^\alpha). \quad (5.17)$$

Given the above normalizations, $\mu_\alpha = \mu_\beta = 1$ and this in turn implies that

$$t^\beta = t^\alpha(1 + z_\alpha). \quad (5.18)$$

However, the fact that the one tax can be set equal to zero implies that $z_\alpha = 0$. From this it follows as above that the shadow prices are also proportional to the producer prices and hence that zero taxes are optimal. However, the multiplier on the government

budget constraint is only identically zero if international transfers are being chosen optimally. Thus, tax harmonization is optimal if and only if the optimal commodity taxes are zero.

6. Conclusion

For the most part, the theory of tax harmonization — and more generally, the theory of taxation in international settings — has assumed the existence of lump sum transfers of resources among nations.¹³ When nations are then assumed to have a single consumer, models tend to feature classical lump-sum taxation for individuals. Tax analysis in this world is very close to the study of first-best taxation. In particular, harmonization arises as a way to alleviate consumer–consumer distortions, a policy that a first-best theorist would advocate. We have shown that such reasoning needs modification when lump-sum transfers are impossible either between or within nations.¹⁴ In general, harmonized tax movements have distributional consequences. At the very least, they may have impacts on the flow of goods and services across borders, with associated disturbance to government revenues. When no mechanism is in place to neutralize these effects, individual countries require extra flexibility in their tax systems in order to maintain budget (and international payments) balance.

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¹³ Exceptions include Turunen–Red and Woodland (1993,1996).

¹⁴ Indeed, this is suggested by Keen (1989) in his concluding remarks.

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APPENDIX

Here, we show that the general model admits three normalizations: one producer price normalization, and one tax normalization per country. We show that adding the constraints

$$dp_1 = dt_1^\alpha = dt_1^\beta = 0 \tag{A.1}$$

has no effect on the system. To do this, we show that their associated multipliers are always zero.

First, rewrite (A.1) in matrix form as

$$\begin{bmatrix} (1, 0_{N-1}^T) & 0_N^T & 0_N^T & 0 & 0 & 0_N^T & 0_N^T \\ 0_N^T & (1, 0_{N-1}^T) & 0_N^T & 0 & 0 & 0_N^T & 0_N^T \\ 0_N^T & 0_N^T & (1, 0_{N-1}^T) & 0 & 0 & 0_N^T & 0_N^T \end{bmatrix} \begin{bmatrix} dp \\ dt^\alpha \\ dt^\beta \\ dm \\ dy^{g\alpha} \\ dy^{g\beta} \end{bmatrix} = 0. \tag{A.2}$$

We check for the existence of feasible Pareto improving directions of reform satisfying the normalization by looking for a solution to (2.28) and (A.2). Second best optima are characterized by their dual system:

$$-v^T + r_\beta \nabla_{y^g}^T F^\beta = 0, \quad (\text{A.3})$$

$$-v^T - z_\alpha p^T + r_\alpha \nabla_{y^g}^T F^\alpha = 0, \quad (\text{A.4})$$

$$\sum_{h \in \mathcal{B}} s_h = v^T \nabla_m \bar{x}^{*\beta}, \quad (\text{A.5})$$

$$\sum_{h \in \mathcal{A}} s_h = v^T \nabla_m \bar{x}^{*\alpha} - z_\alpha (t^{\alpha T} \nabla_m \bar{x}^{*\alpha} - H_\alpha), \quad (\text{A.6})$$

and

$$\sum_{h \in \mathcal{B}} s_h \bar{x}^{*hT} = -v^T \nabla_q \bar{x}^{*\beta} + (\xi_3, 0_{N-1}^T), \quad (\text{A.7})$$

$$\sum_{h \in \mathcal{A}} s_h \bar{x}^{*hT} = -v^T \nabla_q \bar{x}^{*\alpha} + z_\alpha (t^{\alpha T} \nabla_q \bar{x}^{*\alpha} + \bar{x}^{*\alpha T}) + (\xi_2, 0_{N-1}^T), \quad (\text{A.8})$$

$$\sum_{h=1}^H s_h \bar{x}^{*hT} = -v^T \nabla_q \bar{x}^* + v^T \nabla_p \bar{y}^* + z_\alpha (\bar{y}^{*\alpha} - y^{g\alpha T} + t^{\alpha T} \nabla_q \bar{x}^{*\alpha}) + (\xi_1, 0_{N-1}^T), \quad (\text{A.9})$$

where

$$0 \neq s \geq 0, v \geq 0, \text{ and } r \geq 0. \quad (\text{A.10})$$

The normalizations are valid if we can show that the scalars ξ_1 , ξ_2 and ξ_3 are zero at any solution to this system.

First, subtract (A.7) and (A.8) from (A.9) to arrive at

$$0 = v^T \nabla_p \bar{y}^* + z_\alpha (\bar{y}^{*\alpha T} - y^{g\alpha T} + t^{\alpha T} \nabla_q \bar{x}^{*\alpha}) + (\xi_1 - \xi_2 - \xi_3, 0_{N-1}^T). \quad (\text{A.11})$$

Multiplying (A.11) by p , then using homogeneity of supply and the balance of payments condition implies

$$0 = (\xi_1 - \xi_2 - \xi_3) p_1. \quad (\text{A.12})$$

Post-multiply (A.7) by q^β and (A.5) by $-m^\beta$ and then add to give

$$\xi_3 q_1^\beta = 0 \quad \rightarrow \quad \xi_3 = 0. \quad (\text{A.13})$$

Repeating the previous exercise for (A.8) and (A.6) yields

$$z_\alpha (x^{\alpha T} q^\alpha - H_\alpha m_\alpha) + \xi_2 q_1^\alpha = 0 \quad \rightarrow \quad \xi_2 = 0, \quad (\text{A.14})$$

because households in country α satisfy their budget constraints. Now, (A.12), (A.13), and (A.14) imply

$$\xi_1 = \xi_2 = \xi_3 = 0. \quad (\text{A.15})$$

Hence, the three normalization are valid.

It is also easy to see that the same arguments can be used to show that any combination of a single producer price normalization and one consumer price normalization per country proves to be innocuous. For example, the constraints

$$dp_1 = dt_2^\alpha = dt_3^\beta = 0 \quad (\text{A.16})$$

also fail to bind.

Note also that an identical argument, applied to the optimality conditions spelled out in Section 4, gives

$$0 = v^T [\nabla_p \tilde{y}^{\alpha} + \nabla_p \tilde{y}^{\beta}] + (\zeta_1, 0_{N-1}^T), \quad (\text{A.17})$$

where ζ_1 is a multiplier associated with the constraint $dp_1 = 0$ in that model. We note, however, that \tilde{y}^{α} is a function of $p + \tau^\alpha$, while \tilde{y}^{β} is a function of $p + \tau^\beta$. When the prices that the two sets of producers face are different, the matrix in square brackets in (A.17) is, in general, invertible. Thus, either $v = 0_N$ or $\zeta_1 \neq 0$. The former is easily shown to be incompatible with other the other optimality conditions. Thus, the normalization constraint is strictly binding.